Reduction of Warranty Fraud Through Inspection and Combination Warranty Policies for Automotive Batteries

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ABSTRACT

Warranty plays a crucial role in marketing new products as they signal the quality of the product to customers. However, implementing a warranty also requires warranty providers to incur additional costs, known as warranty costs. The annual warranty costs for global automotive manufacturers have increased from 2018 to 2022, with fraudulent warranty claims estimated to account for 3-15% of the total warranty costs. This study aims to develop an effective decision model to minimize warranty fraud on car batteries. Car battery warranty claim data from 2019 was used to develop a failure model, warranty policy models, and decision-making model using Nash game theory. The findings indicate that the Pro-Rata Warranty (PRW) policy is more effective in reducing warranty fraud. The results also suggest that customer behavior in committing fraud is influenced by several factors, including product reliability, penalty costs, inspection costs, battery manufacturing costs, battery purchase costs, and the period of Free Replacement Warranty (FRW) policy. Optimal inspection strategies and appropriate warranty policies can significantly reduce warranty fraud, thereby reducing the warranty cost burden borne by the warranty provider.

Keywords: *car battery, Free Replacement Warranty (FRW), Nash game theory, Pro-Rata Warranty (PRW), warranty fraud*

1. Introduction

Warranty is crucial in marketing new products by signaling product quality to customers. The longer the warranty period, the higher the perceived quality of the product (Murthy & Blischke, 2006). A warranty is a contractual obligation of the manufacturer or seller to handle the repair or replacement of a product during the warranty period (Blischke & Murthy, 2019). The Law of the Republic of Indonesia Number 8 of 1999 concerning Customer Protection mandates manufacturers to provide spare parts and after-sales facilities and to fulfill warranties as agreed.

According to Warranty Week (2023), the annual warranty costs for global automotive manufacturers have increased from 2018 to 2022, amounting to approximately $54.655 billion in 2022. Furthermore, warranty fraud is estimated to account for 3-15% of the total warranty costs (Arnum, 2015 in Kurvinen et al., 2016). Warranty fraud involves dishonest actions by individuals or organizations involved in the warranty process, leading to higher costs or lost revenue for other parties in the chain. Warranty claims fraud is often committed by service agents (SA), customers (C), sales channels (SC), warranty administrators (WA), or the warranty provider (WP) themselves (Kurvinen et al., 2016). Therefore, strategies are needed to address this fraud to reduce warranty costs and minimize fraud.

Previous research by Murthy and Jack (2017) and Pandit and Gupta (2021) discussed warranty fraud by SA towards WP in the form of overbilling on repairable and remanufactured products with a Free Replacement Warranty (FRW) policy using game theory to determine optimal actions of SA and WP and to help WP design the best maintenance service contract with SA. He et al. (2020) also used game theory to determine the optimal actions of SA and WP, where warranty fraud by SA involved providing insufficient service under a fixed-price contract and excessive service under a cost-based contract. This research focuses on warranty fraud by C towards WP regarding car batteries, which are non-repairable products, considering the implementation of Pro-Rata Warranty (PRW) and a combination of FRW and PRW policies.

Car batteries come with a Free Replacement Warranty (FRW), which offers free repair or replacement services if the product fails during the warranty period. This provides an opportunity for C to fraudulently exploit the warranty by obtaining free repair or replacement services, where C uses a battery still under warranty to claim a battery

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whose warranty has expired. From 257 car battery warranty claim data in 2019, 21% or 53 claims were found to be fraudulent. Most frauds occurred at the end of the warranty period, from day 220-380. Fraud control currently involves inspecting warranty claims submitted by customers. However, due to the high cost of inspections, not all claims are inspected, resulting in a high proportion of fraud. To address this issue, this paper proposes the implementation of inspections, penalties imposed on C for fraudulent warranty claims (as applied in the United States (Federal Trade Commission, 1985) and Europe (European Union, 2024)), and a warranty policy requiring customer contribution (such as the PRW policy). Since the replacement of defective products requires customer contribution (not free), it is expected to reduce the proportion of warranty fraud. This research uses game theory to model decision problems for C and WP, aiming to minimize warranty fraud through the implementation of warranty policies, inspection actions, and penalties.

2. Model Formulation

This section will describe the model formulation, which includes warranty policies, failure modeling, revenue modeling, and decision modeling using game theory. Subsequently, the model analysis will be presented to obtain the optimal decision.

Warranty Policy

The common warranty policies provided by warranty providers to customers are FRW and PRW.

1. *Free-Replacement Warranty* (FRW)

Under FRW, if the product fails during the warranty period [0, W], repair or replacement is provided to the customer free of charge (Blischke & Murthy, 2019).

2. *Pro-Rata Warranty* (PRW)

The PRW policy requires customer contribution to cover the cost of repairing or replacing the product if it fails during the warranty period [0, W]. The cost contribution borne by the customer is calculated on a pro-rata basis (depending on the remaining warranty period) (Blischke & Murthy, 2019).

3. Combination of FRW and PRW

This policy combines elements of both FRW and PRW, where FRW is applied during the period [0, W_1], and PRW is applied during the period [W_1, W] with $W_1 \leq W$ being a positive value. FRW allows for repair or replacement without additional costs during the initial period, while PRW offers replacement with costs adjusted proportionally based on the remaining warranty period.

Failure Model

The modeling of car battery failure is conducted to obtain a failure distribution that can represent the failure pattern of car batteries and subsequently estimate the parameters of the distribution. Warranty claim data for car batteries from 2019, was processed using Minitab to determine the best distribution to explain the data pattern. Three commonly used distributions for describing life distribution were considered: Weibull, Lognormal, and Exponential distributions (O'Connor et al., 2016). The results indicated that the Weibull distribution (β=1.99476, α=245.493) was the best fit with the smallest Anderson-Darling value of 8.667. Thus, this Weibull distribution is used to calculate the failure distribution function ($F(t)$) and the failure rate ($r(t)$) as shown in equations (1) and (2):

$$F(t) = F(t;\alpha;\beta) = 1 - exp\left[-\frac{t^{\beta}}{\alpha}\right] \quad (1)$$

$$r(t) = \frac{\beta t^{\beta-1}}{\alpha^{\beta}} \quad (2)$$

Decision Alternatives

Two parties involved in the following decision problem are customer (C) and warranty provider (WP). Each party has two choices when a warranty claim is made. The options for the customer are as follows:

- Option 1 O_{C1}: the customer (C) commits fraud,
- Option 2 O_{C2}: the customer (C) does not commit fraud.

The options for the warranty provider are:

- Option 1 O_{WP1}: the warranty provider (WP) performs an inspection,
- Option 2 O_{WP2}: the warranty provider (WP) does not perform an inspection.

The decision problem facing the two parties is to find optimal options that maximize their respective revenues $J_C^1(p,q), J_{WP}^1(p,q), J_C^2(p,q),$ and $J_{WP}^2(p,q)$ (will be defined later on). The decision variables are the probability of customer fraud ($p \in [0,1]$) and the probability of warranty provider inspection ($q \in [0,1]$). Given these two choices for each party, four possible outcomes arise from their interactions, which are detailed in Table 1.

Table 1. Action Options and Game Outcomes

Warranty Provider (WP)	Customer (C)	
	Commits fraud (O_{C1})	Does not commit fraud (O_{C2})
Performs an inspection (O_{WP1})	O_{C1}, O_{WP1}	O_{C2}, O_{WP1}
Does not perform an inspection (O_{WP2})	O_{C1}, O_{WP2}	O_{C2}, O_{WP2}

Figure 1 illustrates the payoff results for the combination of strategies for each party quantitatively, including the probability of each element and the corresponding payoff. Assumptions in the decision model include that both C and WP are risk-neutral. If C commits fraud and WP does not conduct an inspection, C gains from savings on the purchase and replacement of the battery. Inspections conducted by WP are assumed to be perfect, meaning all fraud is detected. If fraud is detected, C faces a substantial penalty and does not receive a battery replacement. The penalty imposed is significantly larger compared to the costs of inspection and battery replacement borne by WP. The savings C gains from committing fraud are considered to outweigh the inspection costs incurred by WP. Conversely, if C commits fraud and WP does not inspect, WP incurs losses from the costs of battery purchase and replacement.

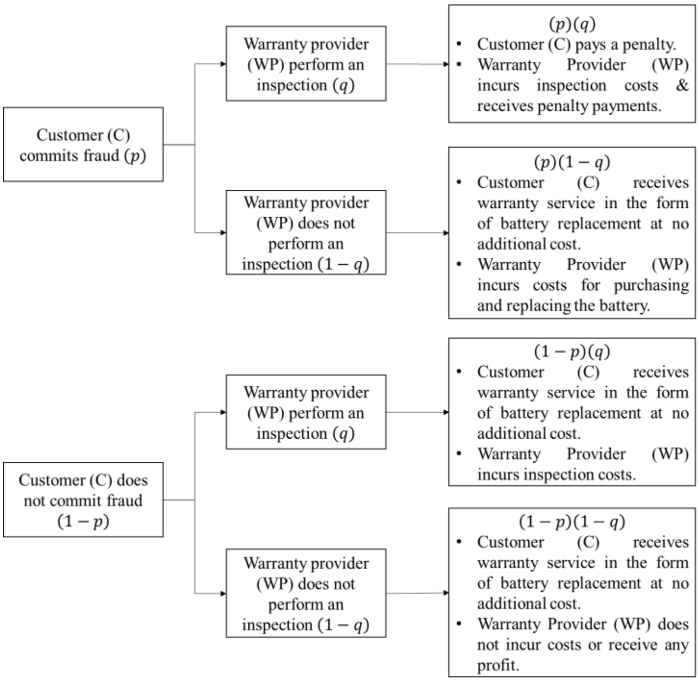

Figure 1. Probability of Each Option and Payoff Results

Revenue Model

This section covers the calculation of warranty costs and then the expected revenue for the considered warranty policies.

1. Expected Warranty Costs with PRW Policy

Assume t is the time of failure (where $t < W$) and S is the cost of purchasing a car battery. The cost borne by the customer is $(1 - \left\{\frac{W-t}{W}\right\} S)$. Thus, the expected warranty cost from the customer's side ($E[C_C^1(W)]$) and from the warranty provider's side ($E[C_{WP}^1(W)]$) are defined as follows. Where δ (a proportional constant) is used to determine the warranty cost for the seller while ensuring reasonable protection for the buyer ($0 < \delta < 1$).

$$E[C_C^1(W)] = \int_0^W \left[1 - \delta \left\{\frac{W-t}{W}\right\}\right] S.dF(t) \tag{3}$$

$$E[C_{WP}^1(W)] = \int_0^W \left[\delta \left\{\frac{W-t}{W}\right\}\right] S.dF(t) \tag{4}$$

2. Expected Warranty Costs with FRW and PRW Combination Policy

When $W_1 = W$, the combined policy becomes an FRW policy. Conversely, when $W_1 = 0$, the combined policy reverts to a PRW policy (Wu & Huang, 2010). A customer contribution is zero if failure occurs during the FRW period $[0, W_1]$. However, if failure occurs during the PRW period $[W_1, W]$, the customer's contribution depends on the age of the product. The expected warranty cost from the customer's side ($E[C_C^2(W)]$) and from the warranty provider's side ($E[C_{WP}^2(W)]$), where Cs is the cost of battery manufacturing, are given by:

$$E[C_C^2(W)] = \int_{W_1}^{W} \left[1 - \delta\left\{\frac{(t-W_1)}{(W-W_1)}\right\}\right] S.\ d\,F(t) \quad (5)$$

$$E[C_{WP}^2(W)] = -CsF(W_1) + \int_{W_1}^{W} \left[\delta\left\{\frac{(t-W_1)}{(W-W_1)}\right\}\right] S\ .dF(t) \quad (6)$$

3. Expected Revenue with PRW Policy

Simply put, the customer's expected revenue is the expected warranty cost from the warranty provider's side. Similarly, the warranty provider's expected revenue is the expected warranty cost from the customer's side. The customer's expected revenue ($E[R_C^1(W)]$) and the warranty provider's expected revenue ($E[R_{WP}^1(W)]$) are given by equations (7) and (8):

$$E[R_C^1(W)] = \int_0^W \left[\delta\left\{\frac{W-t}{W}\right\}\right] S.dF(t) \quad (7)$$

$$E[R_{WP}^1(W)] = \int_0^W \left[1 - \delta\left\{\frac{W-t}{W}\right\}\right] S.dF(t) \quad (8)$$

Assuming P is the penalty cost, and Z_i^1 is the revenue for the customer from the i-th warranty claim (where i = 1, 2, …, $N(W)$) with the PRW policy, where $N(W)$ is the number of warranty claims over W. Considering the probabilities of each element and the payoff (Figure 1), the customer's revenue is given by equation (9):

$$Z_i^1 = \begin{cases} (-P)(pq), & if\ there\ is\ fraud\ and\ inspection \\ R_C^1(t))(p(1-q)), & if\ there\ is\ fraud\ and\ no\ inspection \\ (0)(1-p), & if\ there\ is\ nofraud \end{cases} \quad (9)$$

Assuming I is the inspection cost, and Y_j^1 is the revenue for the warranty provider from the i-th warranty claim with the PRW policy, where $N(W)$ is the number of warranty claims over W. Considering the probabilities of each element and the payoff (Figure 1), the warranty provider's revenue is given by equation (10):

$$Y_j^1 = \begin{cases} (P-I)(pq), & if\ there\ is\ inspection\ and\ fraud \\ (-I)\big((1-p)q\big), & if\ there\ is\ inspection\ and\ no\ fraud \\ -R_{WP}^1(t))\big(p(1-q)\big), & if\ there\ is\ no\ inspection\ and\ there\ is\ a\ fraud \\ (0)\big((1-p)(1-q)\big), & if\ there\ is\ no\ inspection\ and\ fraud \end{cases} \quad (10)$$

The objective functions for the customer ($J_C^1(p,q)$) and the warranty provider ($J_{WP}^1(p,q)$) are given by equations (11) and (12):

$$J_C^1(p,q) = ((-P)(pq)(F(W) + \big(p(1-q)\big).E[R_C^1(W)] \quad (11)$$

$$J_{WP}^1(p,q) = [Ppq - Iq].F(W) + p(1-q).E[R_{WP}^1(W)] \quad (12)$$

4. Revenue Formulation with Combined FRW and PRW Policy

The expected revenue for the customer ($E[R_C^2(W)]$) and the warranty provider ($E[R_{WP}^2(W)]$) with the combined FRW and PRW policy are:

$$E[R_C^2(W)] = \int_{W_1}^{W} \left[\delta\left\{\frac{(t-W_1)}{(W-W_1)}\right\}\right] S.\ d\,F(t) \quad (13)$$

$$E[R_{WP}^2(W)] = -CsF(W_1) + \int_{W_1}^{W} \left[1 - \delta\left\{\frac{(t-W_1)}{(W-W_1)}\right\}\right] S\ .dF(t) \quad (14)$$

Assuming Z_i^2 is the revenue for the customer from the i-th warranty claim with the combined FRW and PRW policy, it is given by equation (15):

$$Z_i^2 = \begin{cases} (-P)(pq), & if\ there\ is\ fraud\ and\ inspection \\ (R_C^2(t))(p(1-q)), & if\ there\ is\ fraud\ and\ no\ inspection \\ (0)(1-p), & if\ there\ is\ no\ fraud \end{cases} \quad (15)$$

Assuming Y_i^2 is the revenue for the warranty provider from the i-th warranty claim with the combined FRW and PRW policy, it is given by equation 16):

$$Y_j^2 = \begin{cases} (P-I)(pq), & if\ there\ is\ inspection\ and\ fraud \\ (-I)\big((1-p)q\big), & if\ there\ is\ inspection\ and\ no\ fraud \\ -R_{WP}^2(t))\big(p(1-q)\big), & if\ there\ is\ no\ inspection\ and\ there\ is\ fraud \\ (0)\big((1-p)(1-q)\big), & if\ there\ is\ inspection\ and\ fraud \end{cases} \quad (16)$$

The objective functions for the customer ($J_C^2(p,q)$) and the warranty provide ($J_{WP}^2(p,q)$) with the combined policy are given by equations (17) and (18):

$$J_C^2(p,q) = ((-P)(pq)(F(W) + \big(p(1-q)\big).E[R_C^2(W)] \quad (17)$$

$$J_{WP}^2(p,q) = [Ppq - Iq].F(W) + p(1-q).E[R_{WP}^2(W)] \quad (18)$$

3. Game Theory Approach

In decision-making scenarios involving two or more decision-makers who may have conflicting objectives, game theory, as discussed by Murthy and Jack (2014), is essential. In some games, decisions are made sequentially, allowing each party to choose actions in a specific order, which is known as Stackelberg game theory. Conversely, there are situations where parties make decisions simultaneously, without knowledge of the other parties' actions, known as Nash game theory. The solution to these games is referred to as the Nash Equilibrium (NE). NE is a set of strategies or decisions for both parties such that no party has an incentive to change their strategy if the strategies of the other parties remain unchanged.

Optimization Procedure

Optimization of the decision problem for the customer (C) and warranty provider (WP) involves the following steps:

- Step 1:
  Customer (C): Determine the value of q (the probability of the warranty provider conducting an inspection)
  Warranty Provider (WP): Determine the value of p (the probability of the customer committing fraud)
- Step 2: Formulate Best Response (BR)

1. Optimization of Decision Problem: [PRW Policy]

The optimization will first be explained from the customer's perspective and then from the warranty provider's perspective.

- Step 1: Customer
  Obtain the value of p, which is the solution to the optimization equation:

$$BR_C^1(q) = argmax\ J_C^1(p,q) \tag{19}$$

By taking the first derivative of $J_C^1(p,q)$ with respect to p and setting it equal to 0:

$$\frac{\partial J_C^1(p,q)}{\partial p} = \frac{\partial\Big((-P)(pq)F(W)+\big(p(1-q)\big)\int_0^W R_C^1(W)dF(t)\Big)}{\partial p} = 0 \tag{20}$$

This result in:

$$q = \frac{E[R_C^1(W)]}{PF(W)+E[R_C^1(W)]} \tag{21}$$

- Step 2: Customer
  To achieve NE, the actions of both players must align with their best responses:

$$BR_C^1(q) = \begin{cases} 1, & if\ q < \frac{E[R_C^1(W)]}{PF(W)+E[R_C^1(W)]} \\ any\ p\ \in [0,1] & if\ q = \frac{E[R_C^1(W)]}{PF(W)+E[R_C^1(W)]} \\ 0, & if\ q > \frac{E[R_C^1(W)]}{PF(W)+E[R_C^1(W)]} \end{cases} \tag{22}$$

- Step 1: Warranty Provider
  Obtain the value of q, which is the solution to the optimization equation:

$$BR_{WP}^1(p) = argmax\ J_{WP}^1(p,q) \tag{23}$$

By taking the first derivative of $J_{WP}^1(p,q)$ with respect to q and setting it equal to 0:

$$\frac{\partial J_{WP}^1(p,q)}{\partial q} = \frac{\partial\Big([Ppq-Iq]F(W)-p(1-q)\int_0^W R_{WP}^1(W)dF(t)\Big)}{\partial q} = 0 \tag{24}$$

This result in:

$$p = \frac{IF(W)}{PF(W)+E[R_{WP}^1(W)]} \tag{25}$$

- Step 2: Warranty Provider
  To achieve NE, the actions of both players must align with their best responses:

$$BR_{WP}^1(p) = \begin{cases} 1, & if\ p > \frac{IF(W)}{PF(W)+E[R_{WP}^1(W)]} \\ any\ q \in [0,1] & if\ \ p = \frac{IF(W)}{PF(W)+E[R_{WP}^1(W)]} \\ 0, & if\ \ p < \frac{IF(W)}{PF(W)+E[R_{WP}^1(W)]} \end{cases} \tag{26}$$

The NE is achieved when:

$$BR_C^1(q^*) = BR_{WP}^1(p^*) \tag{27}$$

$$(p*, q*) = \left\{\frac{IF(W)}{PF(W)+E[R_{WP}^1(W)]}, \frac{E[R_C^1(W)]}{PF(W)+E[R_C^1(W)]}\right\} \tag{28}$$

2. Optimization of Decision Problem: [Combination of FRW and PRW Policies]

The optimization procedure follows steps similar to those for the PRW policy.

- Step 1: Customer
  Determine the value of p, which is the solution to the optimization equation:
$$BR_C^2(q) = argmax\ J_C^2(p,q) \quad (29)$$
  By taking the first derivative of $J_C^2(p,q)$with respect to p and setting it equal to 0:
$$\frac{\partial J_C^2(p,q)}{\partial p} = \frac{\partial\left((-P)(pq)F(W) + (p(1-q))\int_0^W R_C^2(W)dF(t)\right)}{\partial p} = 0 \quad (30)$$
  This result in:
$$q = \frac{E[R_C^2(W)]}{PF(W)+E[R_C^2(W)]} \quad (31)$$
- Step 2: Customer
  To achieve NE, the actions of both players must align with their best responses:
$$BR_C^2(q) = \begin{cases} 1, & if\ \ q < \frac{E[R_C^2(W)]}{PF(W)+E[R_C^2(W)]} \\ any\ p\ \in [0,1] & if\ \ q = \frac{E[R_C^2(W)]}{PF(W)+E[R_C^2(W)]} \\ 0, & if\ \ q > \frac{E[R_C^2(W)]}{PF(W)+E[R_C^2(W)]} \end{cases} \quad (32)$$
- Step 1: Warranty Provider
  Determine the value of q, which is the solution to the optimization equation:
$$BR_{WP}^2(p) = argmax\ J_{WP}^2(p,q) \quad (33)$$
  By taking the first derivative of $J_{WP}^2(p,q)$ with respect to q and setting it equal to 0:
$$\frac{\partial J_{WP}^2(p,q)}{\partial q} = \frac{\partial\left([Ppq - Iq]F(W) - p(1-q)\int_0^W R_{WP}^2(W)dF(t)\right)}{\partial q} = 0 \quad (34)$$
  This result in:
$$p = \frac{IF(W)}{PF(W)+E[R_{WP}^2(W)]} \quad (35)$$
- Step 2: Warranty Provider
  To achieve NE, the actions of both players must align with their best responses:
$$BR_{WP}^2(p) = \begin{cases} 1, & if\ \ p > \frac{IF(W)}{PF(W)+E[R_{WP}^2(W)]} \\ any\ p\ \in [0,1] & if\ \ p = \frac{IF(W)}{PF(W)+E[R_{WP}^2(W)]} \\ 0, & if\ \ p < \frac{IF(W)}{PF(W)+E[R_{WP}^2(W)]} \end{cases} \quad (36)$$
  The NE (p^*, q^*) is achieved when:
$$BR_C^2(q^*) = BR_{WP}^2(p^*) \quad (37)$$
$$(p*, q*) = \left\{\frac{IF(W)}{PF(W)+E[R_{WP}^2(W)]}, \frac{E[R_C^2(W)]}{PF(W)+E[R_C^2(W)]}\right\} \quad (38)$$

Model Analysis

Several theorems have been developed to support the analysis and discussion for testing the proposed hypotheses. These theorems provide proofs that facilitate understanding and evaluation and help identify and explain causal relationships between variables in the study. The developed theorems allow for determining the optimal options that can reduce customer fraud, thereby improving the warranty provider's revenue optimally.

Theorem 1: If the penalty (P) approaches infinity, then the probability of customer fraud (p) and the probability of the warranty provider conducting an inspection (q) approach zero.

Proof: Given by equations (39), (40), and (41).
$$q = \frac{E[R_C^1(W)]}{\infty} \to\ 0 \quad (39)$$
$$p = \frac{IF(W)}{\infty} \to\ 0 \quad (40)$$
$$q = \frac{E[R_C^2(W)]}{\infty} \to\ 0 \quad (41)$$
As $P \to \infty$, then $p, q \to 0$. This means that with a very high penalty, customers are unlikely to commit fraud, and inspections by the warranty provider are no longer necessary. The implication of Theorem 1 is that when penalties are very high, customers tend to avoid fraud, reducing the need for strict inspections or monitoring by the warranty provider. This can help improve operational efficiency and reduce costs since inspections are no longer required in the warranty claims process. The warranty provider might set very high penalties as a strategy to reduce customer fraud incentives but also needs to consider the cost and potential impact of rarely conducted inspections.

Theorem 2: If the penalty (P) approaches zero, then the probability of customer fraud (p) approaches 1, and the warranty provider will conduct inspections (q = 1).
Proof: Given by equations (42), (43), (44), and (45).

$$q = \frac{E[R_C^1(W)]}{0+E[R_C^1(W)]} = 1 \tag{42}$$

$$p = \frac{IF(W)}{0+E[R_{WP}^1(W)]} \rightarrow 0 \tag{43}$$

$$q = \frac{E[R_C^2(W)]}{0+E[R_C^2(W)]} = 1 \tag{44}$$

$$p = \frac{IF(W)}{0+E[R_{WP}^2(W)]} \rightarrow 0 \tag{45}$$

If P = 0, then q = 1, meaning all claims will be inspected. In this situation, customers are more likely to commit fraud due to the absence of penalties. The interpretation of Theorem 2 is that without significant penalties for fraud, there will be a tendency for more customers to commit fraud, increasing the cost burden on the warranty provider. The warranty provider must ensure that the penalties imposed are substantial enough to deter fraud, thereby reducing the need for continuous inspections which can increase operational costs.

Theorem 3: If the inspection cost (I) is very high, then the probability of customer fraud (p) increases.
Proof: From equations (25) and (35), p has a linear relationship with I. The warranty provider is less likely to conduct inspections if inspection costs are high, encouraging customers to commit fraud.

The implication of Theorem 3 is that if the cost of inspections by the warranty provider is very high, the provider is likely to reduce the frequency of inspections. Consequently, customers are more inclined to commit fraud, meaning p will increase. This indicates that inspection costs have a significant impact on the customer's decision to commit fraud. Higher inspection costs increase the likelihood that customers will be motivated to commit fraud. Therefore, it is crucial for warranty providers to carefully consider inspection costs to remain effective in preventing fraud. If inspection costs are excessively high, the warranty provider might need to explore alternative methods for controlling fraud, such as imposing heavier penalties or using cost-effective fraud detection technologies.

Theorem 4: If component reliability increases, the probability of customer fraud (p) will decrease.
Proof: An increase in component reliability (α) implies a decrease in $F(t)$ (equation 1). According to Equations (21) and (31), where $F(t)$ is in the denominator, a decrease in $F(t)$ results in a higher p. Conversely, in equations (25) and (35), $F(t)$ appears in the numerator (multiplied by inspection cost) and the denominator (multiplied by penalty cost). Since the inspection cost is less than the penalty cost ($I < P$), the value of p decreases as $F(t)$ decreases.

The implication of Theorem 4 is that increasing the reliability of components leads to a reduction in the failure distribution function. When components are more reliable, the frequency of component failures decreases, which in turn reduces the number of legitimate warranty claims. With fewer legitimate claims, customers have less incentive to engage in fraudulent activities due to the higher risks or costs associated with fraud compared to the potential benefits. This reduction in fraud has significant implications for warranty providers, including lower costs associated with handling warranty claims and inspections. Investing in component reliability can thus be a cost-effective strategy for reducing fraud and warranty-related expenses. Overall, enhanced reliability not only improves product quality but also helps in managing warranty costs more efficiently.

Theorem 5: If the expected revenue of the warranty provider $E[R_{WP}^1(W)]$ and $E[R_{WP}^2(W)]$ increases, then the probability of customer fraud (p) will decrease.
Proof: From equations (25) and (35), where $E[R_{WP}^1(W)]$ and $E[R_{WP}^2(W)]$ are the denominators, it can be concluded that p decreases if $E[R_{WP}^1(W)]$ and $E[R_{WP}^2(W)]$ increase, meaning customers are less likely to commit fraud.

The interpretation of Theorem 5 is that although the warranty provider's expected revenue might decrease, the probability of customer fraud (p) can increase. This occurs because a decrease in the warranty provider's revenue might lead to policies that financially burden customers, increasing the likelihood of fraud. Conversely, when the warranty provider's expected revenue increases, the provider may be more inclined to implement policies that make customers feel more advantaged, thereby reducing the likelihood of fraud.

4. Numerical Example and Discussion

This section presents the results and discussion related to the implementation of warranty policies to reduce fraud, using numerical examples and sensitivity analysis. Numerical examples were performed to illustrate the optimal solutions under each policy. The cost components used are hypothetical data. The optimal solutions were obtained using Mathcad and Microsoft Excel. The optimization process began with finding the values of p and q as decision variables. Subsequently, the Best Response (BR) or the optimal responses for p and q were determined.

Policy 1 - PRW

The calculations start by determining the failure distribution function, expected revenue for both the customer and the warranty provider, and the optimal values $p*$ and $q*$ using the specified equations. The results for Policy 1 are presented in Table 2. The probability of customer fraud $p*$ is 0.235, and the probability of the warranty provider conducting inspections $q*$ is 0.323. The expected revenue for the customer $J_C^1(p,q)$= 0 and the expected revenue for the warranty provider $J_{WP}^1(p,q)$= 67,455. Figure 2 visualizes the Best Response for Policy 1. The mixed NE ($p*$, $q*$) occurs when $BR_C^1(q^*) = BR_{WP}^1(p^*)$, at the intersection point of both Best Response functions, as shown in Figure 2.

Table 2. Payoff for Policy 1

Probability	q = 0	$q*$ = 0.323	q = 1	
p = 0	0.0	0.0	0.0	$J_C^1(p,q)$
	0.0	-143,875	-444,929	$J_{WP}^1(p,q)$
$p*$ = 0.235	139,697	**0.0**	-292,313	$J_C^1(p,q)$
	116,806	**67,455**	-35,809	$J_{WP}^1(p,q)$
p = 1	595,374	0.0	-1,245,802	$J_C^1(p,q)$
	0.0	258,975	800,873	$J_{WP}^1(p,q)$

Policy 2 – Combination of FRW & PRW

The calculation procedure follows similar steps as for the PRW policy. The results for Policy 2 are presented in Table 3. The probability of customer fraud $p*$ is 0.367, and the probability of the warranty provider conducting inspections $q*$ is 0.184. The expected revenue for the customer $J_C^2(p,q)$= 0 and the expected revenue for the warranty provider $J_{WP}^2(p,q)$= -5,694. The results indicate negative values, meaning that the combination policy of PRW and FRW could lead to financial losses for the warranty provider based on their projected revenue. This implies that the costs incurred for replacing products under this combined policy exceed the revenue generated from product sales or the replacement costs charged to customers. Figure 3 visualizes the Best Response for Policy 2. The mixed (NE) ($p*$, $q*$) occurs when $BR_C^2(q^*) = BR_{WP}^2(p^*)$ at the intersection point of both Best Response functions, as shown in Figure 3.

Table 3. Payoff for Policy 2

Probability	q = 0	$q*$ = 0.184	q = 1	
p = 0	0.0	0.0	0.0	$J_C^2(p,q)$
	0.0	-81,906	-444,929	$J_{WP}^2(p,q)$
$p*$ = 0.367	103,250	**0.0**	-457,624	$J_C^2(p,q)$
	-8,031	**-5,694**	4,663	$J_{WP}^2(p,q)$
p = 1	281,080	0.0	-1,245,802	$J_C^2(p,q)$
	0.0	147,430	800,873	$J_{WP}^2(p,q)$

The value of $p*$ for Policy 1 is lower compared to Policy 2, while the value of $q*$ for Policy 1 is higher. Under Policy 1, the warranty provider is more motivated to conduct inspections because the customer shares the cost of the warranty, which discourages fraudulent claims. In contrast, Policy 2 distributes the warranty cost, potentially reducing the intensity of inspections by the warranty provider and increasing the likelihood of customer fraud. Additionally, the presence of FRW period may incentivize customers to commit fraud. The expected revenue for the warranty provider under Policy 1 is higher compared to Policy 2. This is because, under the PRW policy, replacement of defective products is not provided for free but at a prorated cost. In PRW, customers pay a replacement fee proportional to the benefit they have received from the product before it fails. Meanwhile, the combination policy of FRW and PRW typically offers maximum protection for customers against product failure at the beginning of the warranty period with full replacement or FRW, followed by prorated replacement (PRW) in subsequent periods. The costs associated with full replacement at the start of the warranty period can be substantial and may not be fully offset by the revenues from prorated replacements in later periods, potentially leading to negative expected revenue or losses for the warranty provider (Blischke & Murthy, 2019).

Sensitivity Analysis

Sensitivity analysis is conducted to understand the impact of parameter changes on the optimal decision. The parameters analyzed include scale parameter (α), FRW period (W_1), and delta (δ), as well as cost components such as penalty costs (P), inspection costs (I), and battery manufacturing costs (Cs).

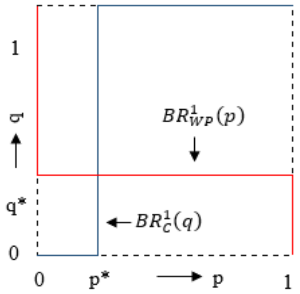

Figure 2. Best Response Functions of Customer and Warranty Provider under Policy 1

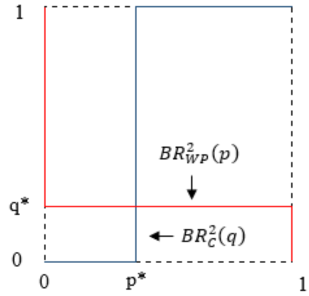

Figure 3. Best Response Functions of Customer and Warranty Provider under Policy 2

A change in the scale parameter or component reliability (α) by 30 days affects the values of p^*. The graph showing the effect of the scale parameter (α) on p^* value (Figure 4) demonstrates that as reliability increases, p^* decrease. This is consistent with Theorem 4, where an increase in component reliability reduces the probability of customer fraud due to fewer warranty claims. A decrease in the delta parameter (δ) by 0.5, 0.6, and 0.7 indicates that an increase in the warranty provider's expected revenue (influenced by δ) leads to a decrease in the probability of customer fraud (p^*), as outlined in Theorem 5 (Figure 5). Changes in the FRW period (W_1) were made up to 90 days and were only calculated under Policy 2. The graph showing the effect of the FRW period (W_1) on p^* (Figure 6) indicates that an increase in the FRW period raises p^*, in line with Theorem 5.

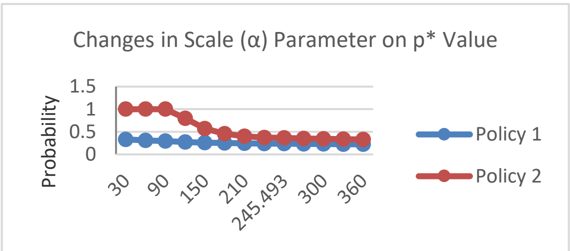

Figure 4. Changes in Scale (α) Parameter on p* Value

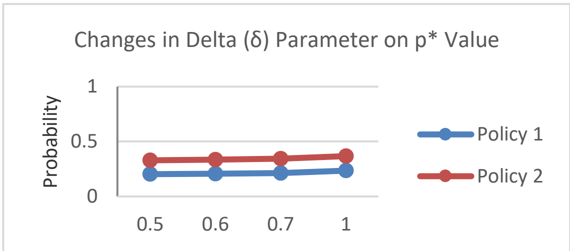

Figure 5. Changes in Delta (δ) Parameter on p* Value

The graphs illustrating the changes in penalty costs (P) on p^* and q^* (Figures 7 and 8) show that increasing penalty costs lowers both p^* and q^*, consistent with Theorems 1 and 2. Conversely, the graph depicting the effect of inspection costs (I) on p^* (Figure 9) shows that increasing inspection costs raises p^*, in line with Theorem 3. The graph showing the effect of battery manufacturing costs (Cs) on p^* (Figure 10) demonstrates that higher manufacturing costs increase p^*, in accordance with Theorem 5. The results of this sensitivity analysis provide a comprehensive understanding of how changes in parameters and cost components impact the optimal decisions in battery warranty policies.

Conclusion

This study developed a decision model for the customer and the warranty provider aimed at minimizing the total warranty costs for car batteries by considering penalty costs, inspection costs, battery manufacturing costs, battery purchase costs, as well as customer fraud probabilities and warranty provider inspection probabilities. The decision model for the PRW policy was represented in equations (21) and (25), while the combined FRW and PRW policy was covered in equations (31) and (35). The findings also indicated that the PRW policy was more effective in reducing warranty fraud for car batteries. This was because, under the PRW policy, customers bore part of the warranty costs, which reduced their incentive to commit fraud. Additionally, warranty providers were more motivated to conduct inspections due to the lower cost burden, enabling more effective detection and prevention of fraud. The effectiveness of PRW in reducing fraud also helped to decrease the total warranty costs borne by warranty providers, including unnecessary inspection and replacement costs.

Future research could enhance the model's accuracy by investing in information technology to improve the precision and speed of warranty claim fraud detection. Furthermore, the implementation of a two-dimensional warranty policy that takes into account multiple variables, such as product age and usage level, could improve model accuracy and assist in identifying inappropriate component use, thereby minimizing false or excessive claims.

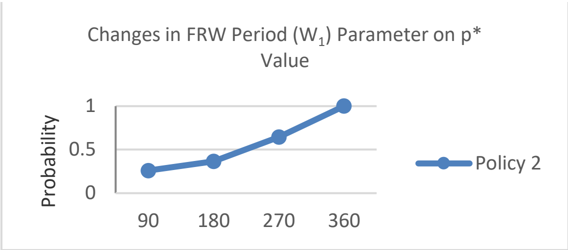

Figure 6. Changes in FRW Period (W_1) Parameter on p^* Value

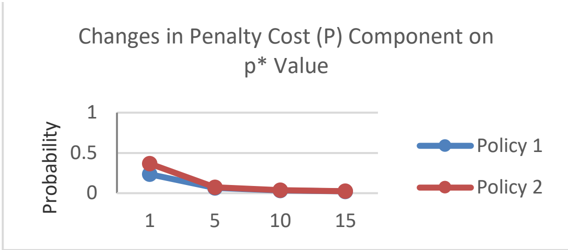

Figure 7. Changes in Penalty Cost (P) Component on p^* Value

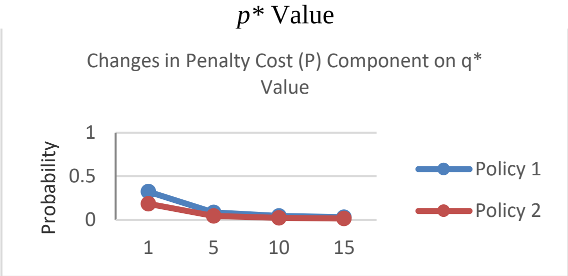

Figure 8. Changes in Penalty Cost (P) Component on q^* Value

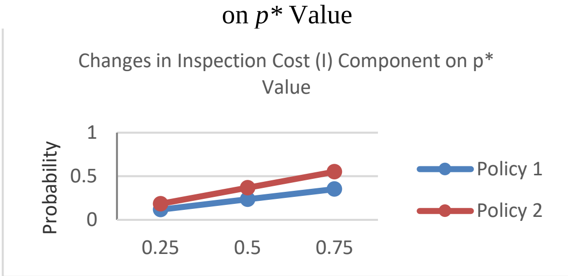

Figure 9. Changes in Inspection Cost (I) Component on p^* Value

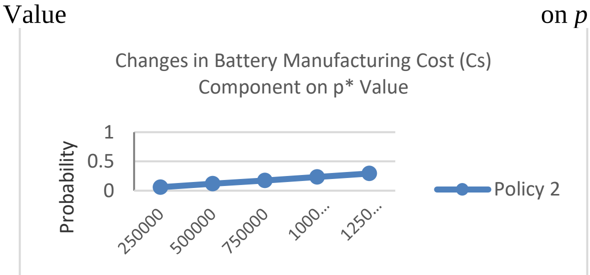

Figure 10. Changes in Battery Manufacturing Cost (Cs) Component on p^* Value

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